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Towards a Rational Constructivist Theory of Cognitive Development

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This article provides a synthesis and overview of a theory of cognitive development, *rational constructivism*. The basic tenets of this view are as follows: (a) Initial state: Human infants begin life with a set of *proto-conceptual primitives*. These early representations are not in the format of a language of thought. (b) Mature state: Human adults represent the world in terms of a set of domain-specific *intuitive theories*. (c) Three types of mechanisms account for learning, development, and conceptual change: language and symbol learning, Bayesian inductive learning, and constructive thinking. (d) The child is an active learner, and cognitive agency is part and parcel of development. I will discuss each of these tenets, and provide an overview of the kind of empirical evidence that supports this view. This is a non-Piagetian view though it is in the spirit of constructivist theories of development; this view emphasizes the utility of formal computational models in understanding learning and developmental change. Lastly, this view also has implications for the study of philosophy of mind and epistemology.

Keywords: rational constructivism, cognitive development, learning mechanisms

> Anything that gives us new knowledge gives us an opportunity to be more rational.
>
> —(Herbert A. Simon)

The study of cognitive development has made great strides in the last few decades. Using the theoretical and empirical tools of developmental psychology and cognitive science, we have developed a much clearer view of how a human infant grows from what was once thought of as "a blooming, buzzing confusion" (William James, 1890/1981) to a highly competent and sophisticated thinker and learner. A theory of cognitive development (and developmental psychology and developmental cognitive science in general) is often characterized as follows: (a) What is the initial state? That is, what does a human learner begin with in terms of her perceptual and cognitive capacities? (b) What is the mature state? That is, where does a human learner end up in terms of her conceptual system as an adult? (c) What are the representations and learning mechanisms that support a learner's progression from a tiny human to a functioning citizen of the modern world? That is, are there representational changes along the way, and what are the mechanisms that drive learning, development, and conceptual change?

This article takes up this challenging task of answering these questions, and provides a synthesis and overview of a theory of cognitive development, namely *rational constructivism.* This theoretical framework is committed to a representational, computational view of the mind (e.g., Chomsky, 1987; Fodor & Pylyshyn, 1988). The central tenets of a rational constructivist theory of cognitive development are as follows (cf. Fedyk & Xu, 2018; Xu, 2007; Xu, Dewar, & Perfors, 2009; Xu & Griffiths, 2011; Xu & Kushnir, 2012, 2013; see Gopnik & Wellman, 2012, for a related but different proposal):

1. The initial representations of a human infant may be best characterized as a set of *proto-conceptual primitives*. These representations are neither perceptual or sensorimotor in the traditional sense (as has been claimed by other constructivist theories), nor fully conceptual (as has been claimed by core knowledge theories). These representations are not in the format of a language of thought (LOT; Fodor, 1975). Equally important, the young human infant has a large toolbox of learning mechanisms that go beyond simple associative learning mechanisms.

2. The mature state of a human learner's mature conceptual system may be best characterized as a set of domain-specific *intuitive theories*, for example, intuitive physics, intuitive psychology, intuitive biology, intuitive sociology, and so forth. These are structured, abstract representations that are theory-like; in the computational cognitive science literature, they are often referred to as *generative models* (see Carey, 1985, 1988, 1991, 2009; Gopnik et al., 2004; Gopnik & Meltzoff, 1997; Gopnik & Wellman, 2012; Leslie, 1994; Spelke, Breinlinger, Macomber, & Jacobson, 1992; Spelke & Kinzler, 2009;

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Tenenbaum, Kemp, Griffiths, & Goodman, 2011; Wellman & Gelman, 1992; Xu & Tenenbaum, 2007a, 2007b). Intuitive theories are causal explanatory frameworks that consist of a set of interconnected concepts and beliefs, expressed as propositional attitudes (i.e., sentences in a natural language).

3. Three types of learning mechanisms drive learning, development, and conceptual change: (a) language and symbol learning, which occurs throughout early childhood and beyond; (b) Bayesian inductive learning, which provides a mechanism for rational belief revision; (c) Constructive thinking, which includes a set of 'learning by thinking' mechanisms such as thought experiment, analogy, explanation, mental imagery, and mental simulation (Gendler, 2000; Lombrozo, 2018). These mechanisms may be characterized as mechanisms for hypothesis generation, conceptual change, and theory change.

4. The child is an active learner. This is the claim that a human learner, from infancy on, is not a passive recipient of information from the environment; instead she plays a critical role in driving her own learning and development (Bruner, 1961; Bruner, Jolly, & Sylva, 1976; Gopnik & Wellman, 2012; Gureckis & Markant, 2012; Piaget, 1954; Schulz, 2012; Singer, Golinkoff, & Hirsh-Pasek, 2006). The child is not just a competent and sophisticated *data processor*, making good use of the input from her environment; she is also at least some of the time a good/rational information seeker or even *data generator*.

The rest of this article explicates each of these claims, and provides an overview of the kind of empirical evidence that supports them. I will end with a few additional discussion points, focusing on the issues of why rational constructivism is a non-Piagetian view, the utility of computational models in the study of cognitive development, some implications for epistemology, and lastly, future directions of research.

The Initial State: Proto-Conceptual Primitives

Much of the study of cognitive development in the last few decades has focused on characterizing the initial state of the human infants. This is unsurprising because it is essential for any discussion of learning and development to begin with an understanding of what infants come to the world with: Has eons of evolution endowed the child with innate concepts and knowledge, and are they uniquely human? Until about the mid-1980s we did not have many methodological tools at our disposal to study the infant mind. With the development of violation-of-expectation looking-time methods, anticipatory looking, eye-tracking, search methods, and imaging methods such as NIRS, ERP, and fMRI, we have made a lot of progress in understanding the infant mind.

Taking stock of an ever-growing, fast-changing body of research on infant cognition, it is perhaps time to rethink how best to characterize the infant mind in terms of the nature of the representations. Two views have been put forth in the literature and discussed extensively. On the one hand, the empiricist view (starting with Hume, 1749/1999; Locke, 1690/1975) has argued for a set of perceptual primitives (e.g., motion detection, color perception, etc.), and the traditional Piagetian view and its contemporary incarnations (e.g., Elman et al., 1996) have focused on sensorimotor primitives—the infant does not have what we think of as representations because whatever these primitives are, they are transient and the infant cannot distinguish her own bodily movements from representations of the external world. Many researchers have argued against this view, and the evidence they have presented is, in my view, quite compelling (see Carey, 2009 for a comprehensive review). Research from the last few decades indicates that infants' cognitive capacity far exceeds what Piaget had granted them, and very young infants' representations of the world persist through time and space (e.g., Baillargeon, Spelke, & Wasserman, 1985; Gelman & Baillargeon, 1983 for a review).

Against this background, a strong nativist view emerged, arguing that infants have a set of "core knowledge" systems at the beginning (Carey, 2009; Carey & Spelke, 1996; Spelke, 1994; Spelke et al., 1992). Here are some prime candidates: object, number, agency, space, and causality. The claim is that infants begin life with these systems of knowledge—designated input analyzers that pick up the relevant entities in the world (e.g., objects, sets, persons, spatial layouts) and a set of principles that guides reasoning about these entities (e.g., object motion, how to perform addition and subtraction, predicting a person's desires and intentions, reorientation). These representations are evolutionarily old, and human infants share them with many nonhuman animals.

But are these representations *bona fide innate concepts*? Critically, how can we tell? Various criteria have been proposed to distinguish between perceptual and conceptual representations. Here I suggest a strong test: Are these early representations in a format that is language-like; that is, are these representations part of a *language of thought* (Fodor, 1975)? The main argument for this strong criterion is that most of us agree that concepts are the mental representations that underlie language use. Adopting this criterion gives us a stringent and principled way of distinguishing concepts from perceptual representations. Note that endorsing this criterion does not entail that nonhuman animals cannot have concepts; they can have them without language, if their concepts are in the right representational format. Thus the question becomes one of whether the early representations have the right sort of format such that they would support language learning in a straightforward way later on. I suggest the answer is *no*. Early representations are computationally and inferentially rich (see evidence reviewed below) but they do not meet the high bar I have set here: They are not in the format of a language of thought. In order to distinguish this view from both the strong empiricist view and the strong nativist view, I will call these early representations *proto-conceptual primitives.* Three main reasons motivate this characterization: (a) Not only do these representations support rich inferences, they support the same inferences throughout development. This is the continuity part. (b) These representations are qualitatively different from later developing representations in their format and whether they support language learning. This is the discontinuity part. (c) These representations cut across the traditional perceptual-conceptual divide, hence the term *proto-conceptual*. Let's take a look at each of the candidate core concepts in turn.

Object

An extensive body of work has revealed a great deal about how infants represent midsized, manipulate-able objects; the significance of the study of the object concept was partly due to the radical claims Piaget had made about how infants lack object permanence therefore mental representations of objects (and therefore mental representations of the world in general). Starting with the seminal work of Baillargeon, Spelke, and Wasserman (1985) and Kellman and Spelke (1983), many studies have shown that infants as young as 4 or 5 months represent objects that are occluded and therefore are not in their visual field directly (see reviews by Baillargeon, 2004, 2008; Spelke, 1990, 1994, among others). These studies initiated the modern study of infant cognition, and opened the floodgate for a myriad of experiments overturning Piaget's characterization of what a human infant's conceptual repertoire is like at the beginning of life. Baillargeon et al. (1985) showed that using the violation-of-expectation looking time methods, 4- and 5-month-old infants understand that when an object is hidden, it continues to exist (the well-known drawbridge experiments, Baillargeon, 1987; Baillargeon et al., 1985). Spelke et al. (1992) laid out a set of principles of how infants reason about objects and object motion: cohesion, continuity, solidity, and contact. Cohesion states that objects move as wholes; they do not spontaneously break into multiple objects or coalescence into a single object. Continuity states that objects move on continuous paths; they cannot move from Point A to point B without traversing a connected path in between. Solidity states that objects cannot occupy the same space at the same time. Contact states that no action at a distance; an object can only act on another object by coming into contact with it. These are beliefs that adults also hold about object motion. Here is how a typical experiment on solidity is conducted using the violation-of-expectation method (Spelke et al., 1992): The infant sat in a high chair facing a small stage. The experimenter was hidden behind the stage, invisible to the infant except for her hand. During habituation, a ball was dropped behind an occluder, then the occluder was removed to show that the ball rested on the floor of the stage. On the test trials, a solid wooden board was placed above the stage floor. The ball was again dropped behind the occluder. When the occluder was removed, the ball either sat on top of the solid barrier (the expected outcome) or on the floor of the stage, below the solid barrier (the unexpected outcome). Four-month-old infants looked reliably longer at the unexpected outcome than the expected outcome. These and other control experiments (e.g., there was a big hole in the solid barrier so the ball could go through) showed that the longer looking times were most likely due to the infants witnessing a highly unusual event—that of a solid object (the ball)—having gone through another solid object (the barrier).

Another extensive body of research by Baillargeon et al. (1985) has discovered that in order to reason about how objects interact with each other, infants divide the world into event categories: occlusion, support, covering, and so forth (see Baillargeon, 2008 for review). For each event category, infants gradually add relevant variables to fine-tune their understanding (e.g., If a tall object were lowered behind a short object, would it be completely occluded? If an object were pushed off of a platform, and its bottom surface continued to overlap slightly with the supporting surface, would it fall to the ground?). Presumably observations in real life and infants' own experiences with manipulating objects later in the first year and beyond gradually allow infants to add the variables of interest for thinking about each event category. The infant's hard won knowledge about objects continues to serve us well as adults.

So where is the discontinuity? Two lines of research point to significant developmental continuities. Several studies have uncovered a surprising failure in older children: When given a task that a 4-month-old infant appears to be able to solve in a looking time version, much older children fail in a comparable search version. Keen, Hood, and their colleagues (Hood, Carey, & Prasada, 2000, 2003; Keen, 2003) adapted the infant solidity experiment in the following simple way: Instead of measuring looking time, 2 1/2-year-old toddlers were asked to find the ball. The apparatus was modified to allow this option. Surprisingly, toddlers searched 50% of the time above the barrier and 50% of the time below the barrier, suggesting that they did not understand that the ball should have been blocked by the solid barrier and therefore should be sitting on top of it as opposed to below it. The children's failure in these search tasks prompted researchers to argue that perhaps the representations revealed by infant looking-time experiments were implicit and relatively weak, such that they did not support prediction and action (e.g., Munakata, McClelland, Johnson, & Siegler, 1997).

A second line of work focused on whether infants reason about objects in terms of sortal-kinds (Carey, 2009; Xu, 1997, 2007). The term *sortal* is borrowed from the study of formal semantics and metaphysics in philosophy. Philosophers have argued that certain concepts, in particular those that underpin our use of count nouns (in languages that make the count/mass distinction), provide criteria for individuation and identity. For example, the sortal *dog* tells us what counts as one dog as opposed to two or three dogs, and it tells us whether we have seen the same dog yesterday, today, and tomorrow. In contrast, a nonsortal such as *water* or *blue* does not provide principles of individuation and identity (see Gupta, 1980; Hirsch, Huberman, & Scalapino, 1982; and Wiggins, 1980 for discussion). This line of philosophical inquiry has inspired much research in cognitive development. A number of studies have focused on whether infants have any criteria for individuation and identity. For example, Xu and Carey (1996) asked whether 10-month-old infants used spatiotemporal or object-kind information for establishing distinct objects in a scene. In one study, infants were shown an occluder on the stage. An object, say a toy duck, appeared from behind the occluder then returned. Next another object, say a ball, appeared from behind the occluder then returned. Critically the infant only saw one object at a time. For adults, our intuition is clear: There are at least two objects behind the occluder, a duck and a ball. What about infants? When the occluder was removed, infants were shown either two objects, the duck and the ball (the expected outcome) or one object, either the duck or the ball (the unexpected outcome). Twelve-month-old infants looked reliably longer at the unexpected outcome than the expected outcome (like adults), whereas 10-month-old infants did not (but see Bonatti, Frot, Zangl, & Mehler, 2002; Futó et al., 2010; and Surian & Caldi, 2010 for earlier success using stimuli that crosses ontological categories such as human vs. nonhuman, or using communicative intent and causal function). An informal survey using a parental checklist found that infants whose parents said they understood at least some of the words that labeled the objects in the experiment—duck, ball, and so forth—succeeded in this task but those who reportedly did not understand the words for these objects failed (Xu & Carey, 1996). Further studies suggest a more nuanced view on object individuation (see Xu, 2007, for a review): Infants are able to use property information (e.g., red vs. green) to establish representations of distinct objects (e.g., Wilcox, 1999; Wilcox & Baillargeon, 1998a, 1998b; Xu & Baker, 2005; Xu, Carey, & Welch, 1999) but not sortal-kind information until the end of the first year (Xu & Carey, 2000; Xu, Carey, & Quint, 2004). For these studies, interestingly, the search methods converged well with looking-time methods (Van de Walle, Carey, & Prevor, 2000; Xu & Baker, 2005).

Despite impressive demonstrations of infant object knowledge, this initial concept of *object* does not appear to be in the format of a language of thought: it does not support the learning of count nouns such as *dog* or *ball.* And there is some evidence suggesting that these representations are not accessible or strong enough to support prediction and action. Instead of calling it the *object concept*, which entails that the concept can support language learning, it is perhaps more appropriate to call it the *object sense* (see next section on the *number sense*).

Number

Developmental psychologists have investigated the origin of the concept of number for decades. This is an important line of inquiry in part because number is an abstract concept that cannot be directly perceived, which raises the question of whether preverbal human infants can represent abstract concepts, and in part because there is a rich literature on how various nonhuman animals represent number and quantity (see Dehaene, 1997/2011; Gallistel, 1990, for reviews). The last 20 years of research has shown, compellingly, that much like many other nonhuman animals, preverbal human infants represent number when other variables such as area, perimeter, density, and so forth are controlled for. These numerical representations are approximate, and they obey Weber's Law—the discriminability of two numerosities is dictated by the ratio of the two quantities, not the absolute difference. Infants can perceive number in multiple modalities, for example, visual or auditory, and the precision of this approximation increases rapidly during the first year of life (see Feigenson, Dehaene, & Spelke, 2004 for review; Brannon, 2002; Lipton & Spelke, 2003; Wood & Spelke, 2005; Xu, 2003; Xu & Arriaga, 2007; Xu & Spelke, 2000). The consensus of the field is that human infants share the same *number sense* with many other animals (Dehaene, 1997/2011). One recent study showed that even newborn infants are able to discriminate between dot-arrays of six versus 18 elements (Izard, Sann, Spelke, & Streri, 2009). Furthermore, infants are able to perform nonsymbolic approximate addition and subtraction on these arrays (McCrink & Wynn, 2004), and preschoolers can even perform nonsymbolic approximate multiplication and division (McCrink & Spelke, 2010, 2016). Thus, rich computations are supported by these analog, approximate representations of number.

The number sense is alive and well in adults, and although the precision of the system changes over time, the signature properties of the system remain the same throughout development. This is the continuity part.

Where is the discontinuity then? The *number sense* supports rich computations, that is, approximate arithmetic, and it is abstract—independent of modalities and perceptual variables that often covary with number (e.g., area). Yet as many have argued, the *number sense* is not a *number concept* (Carey, 2009; Spelke, 2017). The methodology adopted to make this argument echoes what we have discussed in the case of the object concept/sense: Are these numerical representations in the format of a language of thought such that they can support the learning of number words, that is, positive integers such as one, two, three, four, five, and so forth? The answer is clearly "no." Number words appear as an ordered list, and each number represents an exact cardinality, that is, seven means *seven*, not *approximately seven*. The successor function $N_j = N_{j-1} + 1$ makes it clear that the difference between any two numbers on the count list is exactly one, no more and no less. There is also no upper bound to the number list, since we can always add one once we understand the successor function. In contrast, the number sense representations are approximate and imprecise; they follow a psychophysical function that says that the difference between eight and 16 is the same that between 16 and 32; there may be an upper bound on how large of a number the system can represent.

These theoretical and empirical considerations inspired a decade long investigation on how the number sense differs from true number concepts, and how children acquire the concept of *positive integer* and the meaning of number words. To date, the proposals under consideration often posit that verbal counting plays a significant role in this process, and general properties of language (e.g., being symbolic and systematic) help the child build new representations that are genuine number concepts. There is much ongoing debate on how to think about number processing as part of a general magnitude processing system (see Henik, Gliksman, Kallai, & Leibovich, 2017 for a synthesis).

Agency

The infant's world is not only populated by objects they can interact with, but also people that they interact with in a radically different way. A number of research programs has focused on two issues in the development of intentionality and early theory of mind: (a) How do infants identify agents? (b) What do infants endow these agents with in terms of their mental content? Baillargeon, Scott, and Bian (2016) provides a recent review of this literature. The identification of agents relies on not only facial morphology, but also contingent behavior and goal-directed action (e.g., Johnson, Slaughter, & Carey, 1998; Woodward, 1998; among many others). The infant's understanding of an agent is abstract—an agent does not have to look like a person (with a face, eyes, etc.) and it supports rich inferences—if something exhibits goal-directed behavior or if it interacts contingently with the environment, even a blob can be an intentional agent. Adults continue to employ this notion of agency, demonstrating developmental continuity for an abstract sense of agent, *agency sense*.

What is more controversial and currently debated is the mental content of an infant's representation of an agent. Does she have intentions and desires? Does she have beliefs? Some have suggested that the initial understanding of intentions is based on teleological reasoning (Gergely & Csibra, 2003; Gergely & Jacob, 2012; Gergely & Watson, 1999). The infants use their understanding of goals and their understanding of environmental constraints to reason through a situation, and decide what would be the best, most efficient course of action (see Gergely & Csibra, 2003 for an empirical demonstration of this view in 12-month-old infants). More recently, some have suggested that infants as young as 1 year may already have a much more sophisticated understanding of agency in terms of true beliefs and false beliefs.

Onishi and Baillargeon's (2005) seminal article ignited this debate. Using the violation-of-expectancy looking time method, they found that 15-month-old infants were able to compute the consequences of an agent holding a true belief or a false belief. Subsequent studies have extended these results to 7- and 12-month-old infants (Kovács, Téglás, & Endress, 2010). These data present a sharp contrast with the findings of many studies with preschoolers that demonstrated a reliable shift on belief understanding between 3 and 4 years (Wellman, 2014; Wellman, Cross, & Watson, 2001, meta-analysis??, Wimmer & Perner, 1983). With what is known as the Sally-Anne task and many variants of it, verbal false belief tasks have shown over and over again, that across labs, languages, and cultures, it is not until about 4 years of age that children can make correct predictions about an agent's actions. One possibility is that the infant studies reveal some kind of *implicit* understanding of belief and false belief, and there may be a change in the format of children's representations that support the more language-dependent, more *explicit* version of the Sally-Anne task. Setoh, Scott, and Baillargeon (2016) present evidence that with lower processing demand, even 2.5-year-olds may succeed on a verbal version of the Sally-Anne task, but see Rubio-Fernandez et al. (2016) for a critique of these studies—they argue that the training provided to children in Setoh et al. (2016) may have led to correct responses without an understanding of false belief. The empirical results are at present mixed and controversial. The *agent sense* may be another case where both significant continuities and discontinuities exist, and furthermore, the initial representations are abstract and they support rich inferences but language may play a critical role in its further development during the preschool years.

Space

A fourth candidate of initial core knowledge system is space, or geometry. Given the long research tradition on animals' navigation abilities (Gallistel, 1990 for a review), it is natural to ask what the developmental origins are in humans. Has evolution endowed young human learners with navigational tools for surviving in the wild? Do young children use the geometric shape of an environment for orientation and navigation? Do they also use features or landmarks to do so?

Hermer and Spelke (1994) used a classic orientation task, and found that toddlers only used geometry for reorientation. In their study, 18- to 24-month-old toddlers were shown a rectangular room, and a toy was hidden in one of the four corners. After the toddler was spun around a few times and disoriented, she was asked to find the toy. Note that without any distinct features, it is impossible to identify the correct location where the toy is hidden. This is because two of the four corners are geometrically equivalent—they both have a short wall on the left and a long wall on the right; the other two corners are also geometrically equivalent for the same reason. The results showed clearly that most toddlers searched for the toy in the two geometrically equivalent corners, and they did so equally between the two corners. In other words, when the layout of the environment is ambiguous and the best cue is geometry, toddlers use it for reorientation, just like rats and other animals. In the next study, one of the walls was painted blue, thus providing a strong cue that would allow any adult to identify one of the four corners as the correct search location for the hidden toy. Interestingly, toddlers did not use the color of the wall to help them; they continued to search in the two geometrically equivalent corners. Many studies have been conducted since to further probe the characteristics of early geometric/spatial knowledge. Spelke et al. (2010) provides a succinct review of this literature. She argues that these representations—in young human learners and nonhuman animals—are geometric and abstract, because they capture the shape of the environment regardless of surface markings, preserve information on Euclidean distance and left-right direction, and support inferences about the orientation of the self and the locations of objects and places. Furthermore, this system of representations continues to function in human adults, therefore it is developmentally continuous.

The discontinuity comes from several sources. The most relevant for our discussion is the children's failure to use featural information or landmarks for reorientation. Later studies suggest that children begin to use the color of the wall when they start to learn propositions such as "left" and "right" (Hermer-Vazquez et al., 2001), and adults, in a verbal-shadowing task that tied up verbal working memory, perform similarly to toddlers in a reorientation task (Hermer-Vazquez et al., 1999). These data support the idea that language may play an important role in building geometric representations that conjoin geometry and featural information.

However, as Cheng and Newcombe (2005) point out, rats can be trained to use featural cues in a circular or square room, so language is *not* necessary for reorientation with landmarks (Cheng, 1986). Furthermore, human toddlers use the color of the wall for reorientation when they are in a large, but not a small rectangular room. Still, in all developmental studies, the use of featural information, in addition to geometry, increases with age.

For the purpose of the present discussion, it appears that early geometric representations are abstract, as reviewed above, and animal and human learners may primarily rely on the shape of the environment for reorientation and navigation. However, the system is not informationally encapsulated in the strong sense of Fodorian modules (Fodor, 1983). The current state of affairs leaves open the possibility that language and symbol learning may change the format of the early representations in significant ways. The evidence to date provides some support for the idea of a *space sense* that is both developmentally continuous and discontinuous, but it falls short of being a full-fledged concept of space/geometry.

Causality

The last candidate early concept I will consider is causality. An extensive body of research has investigated Michottian causality in adults and infants. This is what we may think of as the canonical understanding of causal interaction: a ball launches and it hits another ball, then the second ball moves. This is the basis of contact causality, a construal that adults continue to hold (Michotte, 1946/1963). A number of infant studies have asked the question: How do infants think about causality, if they have any notion of cause and effect at all? Leslie and Keeble (1987) pioneered the study of causal understanding in preverbal infants. In their study, 6-month-old infants were habituated to a short film in which one rectangle moved toward a second rectangle. The first rectangle either stopped when it comes into contact with the second rectangle, or it stopped short of coming into contact with the second rectangle. Then the second rectangle was launched. On the test trials, both the contact and no-contact films were played in reverse, thus everything else remained the same except that if a causal interaction were perceived (as adults do in the contact scenario), the role of agent and patient (hitter vs. hittee) had also been reversed. Results showed that infants dishabituated more in the case of the reversal of the contact event, suggesting that only the contact event had been perceived as causal, and when the agent and patient reversed their roles, the test event was perceived as more unexpected. Subsequent studies using similar methods find similar results, though in older, 10-month-old infants (Cohen & Oakes, 1993). Recent studies have probed more subtle aspects of adults' causal perception, and demonstrated that infants are also sensitive to these parameters (for some elegant examples, see Kominsky et al., 2017; Muentener & Carey, 2010; Newman, Choi, Wynn, & Scholl, 2008).

Michottian causality is a strong case for an informationally encapsulated module—one can induce the perception of causality with two moving circles on a wall, produced by shining two flashlights, and this perception does not diminish even though adults clearly understand that these circles are not material physical objects. It is developmentally continuous given that we find evidence for Michottian causality in infants and adults.

This early developing sense of causality (the *causal sense*) may be contrasted with the modern interventionist view of causality (e.g., Danks, 2014; Gopnik et al., 2004; Woodward, 2003). This view of causality suggests that learners track the conditional probabilities of events and use these computations to infer causal relationships. Moreover, the critical criterion is *difference making*: Learners are granted an understanding of causality when they are able to produce effective interventions, for example, if I intervened on Node A of the causal graph, then Effect B would not occur any more. Several studies with young children suggest that producing effective interventions may require causal language (Bonawitz et al., 2010; Muentener, Bonawitz, Horowitz, & Schulz, 2012).

Causality also poses a different challenge from our previous discussions about object, number, agency, and space. Each of these four core knowledge systems focuses on a particular content domain. Causality, however, is central for each system but manifests itself differently. For infants, the evidence for Michottian physical causality is strong; in contrast, understanding intentional causality seems to require different considerations. For example, Muentener and Carey (2010) found that 8-month-old infants construed state change (e.g., a box being crushed into pieces) as causal only when an intentional agent was involved. They suggest that there may be two developmental pathways for early causal understanding—Michottian physical causality and effective intentional action. It may be the case that causality is best understood as a domain-specific construct early on, then the interventionist view may be acquired via language so that eventually all causal understanding includes components of tracking conditional probability, prediction, and intervention (e.g., Bonawitz et al., 2010).

Further support for the idea that these early representations may sit comfortably at the perceptual-conceptual interface, hence the term *proto-conceptual primitives*, comes from a body of work on perception and visual cognition. Scholl and his colleagues have demonstrated, convincingly in my view, that many of the core knowledge system signatures are represented in the perceptual system. For example, in a series of experiments with adults, Chen and Scholl (2016) probed whether perception of causal history may induce illusory motion perception. Participants watched a square change into a truncated form, with a piece of it missing. When the contours of the missing piece suggest a history of intrusion (akin to poking a finger into clay), participants reported seeing gradual change even when it was actually abrupt. In other words, visual perception involves reconstructing causal history from static shapes. Thus a high-level concept such as *causality* is seamlessly integrated into automatic, rapid visual perception. A second example investigates whether cohesion violations (part of core knowledge of object, see previous section on object) disrupt multiple object tracking, a standard task for studying object perception. Scholl, Pylyshyn, and Feldman (2001) asked participants to track several objects in a field of identical distractors. When a target object and a distractor are "merged" (e.g., being connected by a line between them), participants' ability to track the object was disrupted and performance dropped significantly. Similarly, found that participants can track objects as coherent units in a multiple object tracking task, but they had great difficulties tracking piles of substances that are "poured" from location to location. Thus, a cohesion violation (e.g., in tracking piles of sand in real life) disrupts tracking of objects in midlevel vision. A third finding reinforces this conclusion. Mitroff, Scholl, and Wynn (2004) found that when an object splits into two (a cohesion violation), object tracking and the benefits of preview object files are interrupted, incurring significant cost in performance. Taken together, these findings strongly support the idea that many aspects of the core knowledge systems are represented in our midlevel visual perception system (see also Carey & Xu, 2001; Leslie, Xu, Tremoulet, & Scholl, 1998; Xu, 1999 for similar proposals).

Why Are These Early Representations Proto-Concepts and Not in the Format of a Language of Thought?

From the evidence reviewed above, it appears that these early representations are domain-specific and inferentially rich (Carey, 2009; Spelke, 1994). At the same time, these representations are "fragile" in multiple ways. These representations may be largely automatic and informationally encapsulated (though not as strictly as Fodor suggests); these representations may be implicit and not robust enough to support prediction and action; these representations may not be in the right format to support language learning; and these representations appear to be subject to some of the same constraints on midlevel perceptual/attentional systems.

I have suggested for a rather stringent criterion for concepts, that the representations under discussion should be in the format of a language of thought such that they can support language learning, be it words or other aspects of language. Furthermore, a language of thought—perhaps a natural language—is needed for formulating beliefs in our intuitive theories, and for going beyond perceptual/iconic representations. I make two further points in this section, and I hope this will be the beginning of a more extensive discussion of these issues for all students of cognitive development.

First, in contrast to the criterion for concept-hood I suggest above, the extant literature offers two other criteria for concepts: One is to examine whether these representations are central and amodal (Spelke et al., 1992), and the other is to examine whether these representations support rich inferences (Carey, 2009).

Spelke's proposal was put forth against the background of the traditional Piagetian view, that early representations are sensorimotor representations. She argued convincingly that the initial construal of object is not sensorimotor, and it is amodal given evidence of visual and haptic matching experiments (Streri & Spelke, 1988). Our discussion of early numerical understanding reinforces this point: Young infants are able to compute approximate numerosities in visual and auditory modalities. Yet our discussion on both object and number above suggest that this criterion may be too weak.

Carey's proposal was put forth with a general theory of concepts in mind, that concepts are defined by their inferential role in a system of interconnected concepts and beliefs (Block, 1986; Carey, 1985, 2009). I do not doubt the importance of understanding inferential role in a theory of concepts, but a key problem is that Carey may have underestimated the inferential capacity of our perceptual system. A myriad of examples in early vision (e.g., Feldman, 2012; Feldman & Singh, 2006; Knill & Pouget, 2004; Knill & Richards, 1996; Weiss, Simoncelli, & Adelson, 2002) suggest that the perceptual system is capable of very sophisticated and subtle inferences. Many have suggested these inferences are best captured with probabilistic Bayesian models, and the visual system rationally combines prior constraints and biases with input from the environment (See Knill & Richards, 1996, for a review).

An important caveat is in order here. I have advocated that to examine whether certain representations are in the format of a language of thought provides the litmus test for whether we have a bona fide *concept*. This is meant to be a *methodological* suggestion, so we can set a clear and principled criterion for concept-hood, but it does not entail that learners cannot have concepts without language. Nonhuman animals can, in principle, acquire the later, more mature versions of these early concepts via other nonlinguistic mechanisms. At the same time, I will argue that for human learners, language and symbol learning is a crucial mechanism for developmental change (see section on language and symbol learning below).

Second, we can directly ask the question "Do prelinguistic children have a language of thought?" and test it empirically. Some key properties of a language of thought have been laid out clearly by Fodor and others (Fodor, 1975; Fodor & Pylyshyn, 1988): systematicity and compositionality. A few recent studies have begun to probe these properties empirically, and the results seem to suggest that no, prelinguistic infants and toddlers do not have a LOT. For example, Piantadosi, Palmeri, and Aslin (2018) and Piantadosi and Aslin (2016) asked whether 9-month-old infants could anticipate the outcome of two functions, that is, not just f(x) and g(x), but also g(f(x)), a strong test of compositionality. They taught infants that two separate "functions," for example, if a blue object goes behind an occluder, it comes out red (color change); if a polka-dotted object goes behind a different occluder, it comes out striped (pattern change). The critical question if what infants think would happen when an object goes behind the first, then the second occluder—do they expect two changes (color and pattern)? The answer appears to be no. In other words, infants had failed to compose two functions, even though control experiments showed that they were capable of learning each function separately. Another line of research asks whether logical operators such as "no"/"not" are understood as genuine operators that change the truth-value of a proposition, since for adults, they do both in our language of thought and in our language. Austin, Theakston, Lieven, and Tomasello (2014) and Feiman, Mody, Sanborn, and Carey (2017) found that it is not until 24–27 months of age that toddlers start to use "no/not" as logical operators and make inferences accordingly. Thus far, we do not yet have evidence that representations in prelinguistic infants demonstrate key properties of a language of thought.

In sum, this section argues for the thesis that very early representations are neither perceptual nor conceptual, and we dub them *proto-conceptual primitives*. These representations may be amodal and they may support rich inferences; however, they fall short of meeting the criterion of having a format that is in a language of thought (Fodor, 1975), therefore they are not genuine *concepts*.

The Mature State: Domain-Specific Intuitive Theories

The mature state of cognitive development has been characterized by many as a set of domain-specific *intuitive theories* (e.g., intuitive physics, intuitive psychology, intuitive biology, intuitive astronomy; Carey, 1985, 1988, 2009; Carey & Spelke, 1996; Gopnik, 1996; Gopnik & Meltzoff, 1997; Leslie, 1994; Vosniadou & Brewer, 1994; Wellman, 1990; Wellman & Gelman, 1992). Wellman and Gelman (1992) call these "framework theories"—they are "intuitive theories that carve phenomena into differing organized systems of knowledge and belief." These knowledge structures are "theory-like" in important respects (Carey, 1985, 1991; Carey & Spelke, 1996; Gopnik, 1996; Gopnik & Meltzoff, 1997; Spelke, 1991). First, the content of these knowledge structures is organized in terms of a set of causal, explanatory concepts and beliefs, much like scientific theories. The concepts are mental tokens that gain their explanatory power by being connected via beliefs (e.g., all living things grow and reproduce). Second, these knowledge structures undergo two types of changes, belief revision and conceptual change, much like scientific theories. Belief revision is common, given the accumulation of factual knowledge (e.g., a child may not know that all insects have six legs initially but acquire this knowledge from their parents or in school). Human learners have well-oiled machinery for everyday belief revision (see Bayesian Inductive Learning as a Tool for Rational Belief Revision section below). Conceptual change occurs more gradually, partly based on cumulative belief revision and partly based on changes in causal understanding. For example, consider the case of intuitive biology: A child may not understand that animals and plants are all living things initially. When animals and plants are subsumed under the global category of "living things," much of her biological knowledge—breathing, growth, reproduction, and so forth—changes as well. And when we consider the case of intuitive physics, we see other types of conceptual change with far-reaching consequences. For instance, preschoolers have undifferentiated concepts of weight and density, because of their construal of size, matter, and material kind. By the time children are 8 or 9 year of age, they split the concept weight/density into two distinct concepts, and work out the relation between the two, all through laborious thinking and reasoning (Smith, Carey, & Wiser, 1985). Scientific theories evolve and change in similar ways. Furthermore, there is some developmental evidence supporting the contention that belief revision and conceptual change are indeed distinct processes; a learner may accumulate many biological facts without understanding the underlying the causal explanatory mechanisms (Johnson & Carey, 1998).

For intuitive physics, infants begin with the principles of reasoning that Spelke and her colleagues have laid out clearly (e.g., Spelke, 1994; Spelke et al., 1992). Later on, children develop a theory of matter that includes core concepts such as mass, weight, density, solid and nonsolid substances, gases, and object kinds (Smith et al., 1985).

For intuitive psychology, infants begin with the principles of intentionality (e.g., Gergely & Csibra, 2006; Wellman, 1990) to reason about goal-directed action (e.g., Woodward, 1998). Later on, at least according to some researchers, children develop a theory of mind that includes a more robust understanding of true and false beliefs, deception, lying, and other high-level mental constructs (e.g., Chandler, Fritz, & Hala, 1989; Gopnik & Meltzoff, 1997).

For intuitive biology, infants may start with a very rudimentary and skeletal understanding of animacy based on self-generated movement. Later on, children develop a vitalistic biology (e.g., Inagaki & Hatano, 2004; Slaughter, Jaakkola, & Carey, 1999; Slaughter & Lyons, 2003) that is based on the core idea of life force (or "qi" in Asian cultures). Some also develop a more mechanistic understanding of bodily functions, the life cycle, and reproduction.

As Carey and Spelke (1996) pointed out, these knowledge structures are fundamentally different from core knowledge systems (in our terminology, *proto-conceptual primitives*). Core knowledge systems identify their entities for reasoning via perceptual analysis, whereas the entities reasoned about in intuitive theories are identified in much more theory-laden ways. Core knowledge systems are shared by nonhuman animals and evolutionarily old, whereas presumably the capacity for constructing intuitive theories is uniquely human. Core knowledge systems are response- and task-specific, whereas intuitive theories are central reasoning systems that can be widely used to support prediction and action. Lastly, core knowledge systems are constant throughout development, with minor revisions and elaborations, whereas intuitive theories are open for revisions and radical conceptual change, so much so that children and adults' later theories may be incommensurable with their earlier ones.

Another important issue to consider is whether domain-specific intuitive theories grow out of early representations, and whether they are restricted to the domains represented by proto-conceptual primitives or core knowledge systems. The answer seems to be no. On the one hand, intuitive physics has at its core the concept of *object* and intuitive psychology the concept of *agent*, but human adults do not appear to develop an intuitive theory of navigation or domain-general causality. As for the domain of number, human adults continue to use the number sense in a variety of ways, and we learn to do arithmetic and in some cases high-level mathematics, but it is unclear whether we have a *theory* of number. In contrast, some have argued that learners form intuitive theories of new domains such as intuitive chemistry (e.g., Au, 1994), intuitive astronomy (e.g., Vosniadou & Brewer, 1994), and intuitive theory of morality (e.g., Rhodes & Wellman, 2017). The key point here is that intuitive theories are not continuous with proto-conceptual primitives, neither in format nor in content.

This discussion naturally raises the question about mechanisms: What are the mechanisms of learning that drive the construction of intuitive theories, if human infants begin with a set of proto-conceptual primitives?

Mechanisms of Learning, Development, and Conceptual Change

I have argued that human infants begin life with a set of *proto-conceptual primitives* and by middle childhood, they have developed a set of *domain-specific intuitive theories*. These proto-conceptual primitives straddle the perceptual-conceptual boundary, and they are not in the format of a language of thought; the later-developing intuitive theories are genuinely conceptual representations and they are expressed by propositions in language. If our characterization of the initial state and the final state is on the right track, then we need mechanisms for three types of changes. First, mechanisms that transform the proto-conceptual primitives into a format that is conducive for language learning and reasoning by propositions (i.e., a language of thought). Second, mechanisms for belief revision since intuitive theories are consisted of a set of concepts and beliefs, and our beliefs change in the face of evidence. Third, mechanisms for genuine conceptual change—even our core concepts and core beliefs embedded in intuitive theories may be radically revised, and we may undergo theory change that includes the possibility of incommensurability (e.g., Carey, 1985; Gopnik & Meltzoff, 1997).

Language and Symbol Learning as a Medium for Changing the Format of Early Representations

Language learning starts in utero. Newborn infants show a preference for human speech over other auditory stimuli (e.g., Vouloumanos, Hauser, Werker, & Martin, 2010), and they already prefer their native language (or any other language that is in the same rhythmic class as their native tongue) over a non-native language. Speech perception develops rapidly during the first year of life (e.g., Werker & Hensch, 2015; Werker & Tees, 1984), and word segmentation and word learning begin in earnest from about 6 months on (e.g., Aslin, Saffran, & Newport, 1998; Saffran, Aslin, & Newport, 1996). Toddlers and preschoolers can use syntactic cues to narrow down noun and verb meanings (e.g., Berko, 1958; they begin to understand absent reference as early as 12 months and they use language—in the form of testimony—for learning many aspects of domain-specific knowledge from very early on (Koenig & Harris, 2007). Language is the first symbolic system that children acquire, but it is not the only one. Many children are exposed to Arabic numerals, maps, and other symbolic forms quite early. Does language and symbol learning change initial proto-conceptual primitives into representations in the format of a language of thought, such that further learning and reasoning is carried out in words, symbols, and propositions? In this section I put forth a working hypothesis that learning a natural language serves as the medium of building LOT-like, genuinely conceptual, representations.

Language and the object sense. Research from the last decade shows that as early as 3 to 6 months, infants can use the presence of a count noun label to facilitate the formation of an object category (see Perszyk & Waxman, 2018 for review); by 9 months, they use words but not tones in forming categories; by 14 months, they use count nouns to form superordinate categories (whose members do not share many perceptual features) and adjectives for identifying attributes or properties of objects (e.g., spotted). Also during the first year of life, words begin to facilitate infants' individuation of distinct sortal-kinds (Dewar & Xu, 2009; Xu, 2002; Xu & Carey, 1996; Xu, Cote, & Baker, 2005), but not properties such as color (Xu et al., 2004).[1] Furthermore, the presence of a count noun helps 10- to 13-month-old infants make inductive inferences about nonobvious object properties, even when the objects are perceptually identical (Baldwin & Markman, 1989; Dewar & Xu, 2009; Welder & Graham, 2006). These three lines of research provide converging evidence that within the first year of life, infants already make good use of the ambient language around them to build a better conceptual and representational structure of the world. Language helps infants group objects into categories that do not share many perceptual features; language helps infants distinguish between distinct object categories and kinds; language licenses inductive inferences of nonobvious object properties even when there is little perceptual overlap across a set of exemplars (a version of psychological essentialism, Gelman, 2003). As early as 3 months, words are referential symbols that go beyond perceptual similarities and infants use them to form beliefs about not just individual objects but whole categories of objects (see Sloutsky, 2009 for a different view of the role of words and concepts, and Waxman & Gelman, 2009, for a rebuttal).

Language and the number sense. Several proposals are in the literature on how children acquire the meaning of numbers words such as one, two, three, four, five, and so forth. Each of them puts a heavy emphasis on language learning, either the counting routine or the noun phrases children learn. One view suggests that children begin by memorizing the counting list, simply repeating by rote the words one, two, three, and so forth without knowing the semantics of these words (this is a well-established empirical finding, e.g., Le Corre & Carey, 2007; Sarnecka, Kamenskaya, Yamana, Ogura, & Yudovina, 2007; Wynn, 1990, 1992). Toddlers learn the distinction between singular and plural marking in a language like English, which corresponds to the distinction between one object and more than one object. This provides the basis for understanding that all number words refer to quantities. The child works through the counting list slowly, first learning the meaning of "one" as referring to one object, then "two" as referring to two objects, then "three" as referring to three objects. Occasionally a child may continue to "four" in this manner. The meaning of these number words is grounded in the parallel individual system (or object files, Kahneman, Treisman, & Gibbs, 1992; Scholl & Pylyshyn, 1999). But the learner only has three object files, so when she encounters the number word "four," more conceptual work is needed. The idea is that the child now makes an inductive leap—she notices that from one to two to three, each time she adds one object file to create the corresponding nonlinguistic representation of the corresponding set. By analogy, when it comes to four or five or six, she will add one (imaginary object file) in order to represent the cardinality of the set (Carey, 2009, 2014; Le Corre & Carey, 2007; Sarnecka et al., 2008; Wynn, 1992). This proposal relies on the idea that the child assumes a counting list that consists of number words all have the same kind of meaning, so the successor function of "adding one" will have the inductive force needed. A second proposal suggests that this inductive learning process goes more slowly, and relies on yet another aspect of language learning, namely learning the meaning of noun phrases such as "three tigers" or "four flowers" (Spelke, 2017). This proposal is motivated in part by the empirical finding that in a training study, children learned the meaning of the number word "three" in the context of a picture showing three tigers, but they did not generalize the meaning of three to other pictures showing three flowers or three bricks. The alternative view suggested is that children acquire partial meaning of number words by embedding them within noun phrases. Gradually, a second inductive process takes places and children generalize the meaning of "three" to all instances in which the cardinality of the set is exactly three. Further support for the idea that language plays a critical role in representing large, exact numbers comes from a study with adults (Spelke & Tsivkin, 2001), in which bilingual speakers were more effective in retrieving information in the language of training, with exact numbers but not with approximate numbers.

Both of the proposals discussed above argue for a critical role for language learning, whether it is the counting list or the meaning of noun phrases. It may be the case that learning the counting list provides an external tool for making the analogy between object files and number words. However, there is also a stronger possibility: That it is language (learning the meaning of number words, more specifically) that fundamentally changes how young human learners represent number. Infants only have the fuzzy, approximate numerical representations; these representations obey Weber's Law such that the difference between eight and 16 is larger than the difference between 108 and 116. It is via language learning that children come to represent numbers as discrete, symbolic, abstract entities that are governed by a formal recursive rule: The successor function that dictates that "add 1" captures the relationship between any two successive number words on the list, and the difference between eight and 16, and 108 and 116, is exactly the same, eight. That is, the counting list provides a crucial medium for the change in representational format, and this leap is the beginning of a mathematical ability that is beyond the reach of nonhuman animals, individual home sign users, and nonindustrialized traditional societies (e.g., Gordon, 2004; Jara-Ettinger, Pi-

[1] Bonatti et al. (2002) and Surian and Caldi (2010) presented evidence that before 12 months, infants may already represent *human* or *person* as a sortal concept. They found that when given an agent versus non-agent contrast, 10-month-old infants were able to use that difference to establish a representation of two objects under occlusion, and when given two agents, infants were not able to do so. These findings provide an exact parallel to the Xu et al. (2004) findings with inanimate objects. Now let's consider how these representations support the learning of count nouns, the linguistic counterpart to sortal concepts. For sortals such as *ball* and *truck*, the story is straightforward. We suggest that at 9 months, infants use the words "ball" and "truck" to help them build representations of sortals, and by 12 months, they have these representations of sortals, but not properties (red ball vs. blue ball). For agents, the story is interestingly puzzling. The Bonatti and Surian studies suggest an early representation of the sortal *person*, but not *Joe* versus *Mary* (two distinct agents/individuals), but when we look at infants' early lexicon, proper nouns that refer to unique individuals are among the first they acquire, *Mama* and *Dada*, *Max* (the family dog), and it is a long while before children learn the word *person*. This raises interesting questions about how to relate pre-linguistic representations and early word learning.

antadosi, Spelke, Levy, & Gibson, 2017; Saxe, 2015; Spaepen, Coppola, Spelke, Carey, & Goldin-Meadow, 2011;).

Language and the agent sense. It is clear from the previous discussion on early understanding of agency and mental states that right now there is no consensus in the field on how to characterize the 1-year-old's theory of mind, and whether there is genuine theory change at around 4 years of age. One possibility is that the looking time tasks reveal an implicit understanding of belief early on (Kovács et al., 2010; Onishi & Baillargeon, 2005), but to succeed at the verbal version of the classic false belief task, a stronger and more explicit representation is needed (Wellman, 2014). This understanding may come about with using propositions to represent beliefs, and reason logically through a set of premises in order to draw the correct conclusions. de Villiers and colleagues, in a provocative study, found that deaf children with hearing parents were delayed in false belief and knowledge state tasks, whereas their nonlanguage-delayed counterpart, deaf children with deaf parents were not. These findings provide some support for the idea that language skills may be important in developing theory of mind in preschoolers, and de Villiers and colleagues argued for a bidirectional account (de Villiers, 2007; Schick, de Villiers, de Villiers, & Hoffmeister, 2007). Other studies have also provided data supporting the idea that language is critical in theory of mind development, in both longitudinal studies with typically developing children as well as children with language delay (e.g., Astington & Jenkins, 1999; Pyers & Senghas, 2009;Ruffman, Slade, & Crowe, 2002). The key findings are that first, children's theory of mind development is predicted by their language development, not vice versa; and second, children with language delay or impoverished language input are also delayed in theory of mind development. There is no question that this is a controversial claim (see Carruthers, 2011, 2016, and others for discussion).

Language and the space sense. Although studies with nonhuman animals provide compelling evidence that language is *not necessary* for using featural information of the environment for reorientation (Cheng, 1986; Cheng & Newcombe, 2005), the body of research reviewed above leaves the door wide open that language may indeed play a critical role in human learners' development of reorientation, navigation, and geometric understanding.

Hermer-Vazquez, Moffet, and Munkholm (2001) found that 5- to 7-year-olds who knew the meaning of words like "left" and "right" were better able to combine geometric and featural information in a reorientation task. Hermer-Vazquez, Spelke, and Katsnelson (1999) found that when adults were engaged in a verbal-shadowing task, their reorientation relied more on geometric information at the expense of featural information, much like toddlers. However, these findings are quite controversial (see Cheng & Newcombe, 2005; Ratliff & Newcombe, 2008).

Language and the causal sense. The Michottian causal sense is a strong candidate for a perceptual module—it is automatic and encapsulated; it also continues to function in adults. What about the interventionalist view of causality (Danks, 2014; Gopnik et al., 2004; Gopnik & Schulz, 2004; Woodward, 2003)? As I reviewed above, it is unclear whether causality is a domain-general primitive (see Carey, 2009, for arguments on the affirmative side). And as discussed earlier, causality may be best understood as a domain-specific construct given its importance in reasoning within each domain, be it about object or agents. The interventionist view may be acquired via language— causal language serves as a unifying force—such that eventually all causal understanding includes components of tracking conditional probability, prediction, and intervention. Results across several laboratories provide strong and consistent empirical evidence that young children use causal language to shape their causal understanding both through testimony (e.g., Harris & Koenig, 2006) and through labeling (Gopnik, Sobel, Schulz, & Glymour, 2001; Nazzi & Gopnik, 2001).

In sum, in each case of early representations, be they about objects, numbers, space, agency, or causality, there appear to be discontinuities between the initial representations of infants and the more mature concepts held by older children and adults. In each case, there appears to be some evidence for language and symbol learning to play a critical role in development, although there is also a fair amount of controversy surrounding these findings and arguments. One unifying possibility, building on the argument that early representations are not in a format of language of thought, is that as language and symbol learning progresses, learners acquire the ability to express all early representations in a format that is compatible with language. That is, there is an across the board change in representational format (Karmiloff-Smith, 1990), and these new representations are propositional, and accord with general principles for a language of thought such as compositionality and systematicity (e.g., Fodor & Pylyshyn, 1988; Lake, Salakhutdinov, & Tenenbaum, 2015; Pinker & Prince, 1988). This is indeed the beginning of the construction of domain-specific intuitive theories.

Bayesian Inductive Learning as a Tool for Rational Belief Revision

The study of cognitive and language development has long been integral to the study of cognitive science—perhaps unsurprising since a big part of understanding human cognition is to understand its origin and development, and to understand the nature of the learning mechanisms. A significant part of this integrated approach appeals to the use of computational models as a formal tool for understanding learning and development. Many have employed connectionist neural network models to capture the emergent properties of development, therefore these models have often been seen as instantiations of certain constructivist ideas (e.g., Elman et al., 1996; Munakata et al., 1997). The last decade or so has witnessed a surge of probabilistic Bayesian models as a tool for understanding human cognition, learning, and development. Bayesian models are distinct from neural network models in a number of important respects (see the exchange between McClelland et al., 2010 and Griffiths, Chater, Kemp, Perfors, & Tenenbaum, 2010; Lake et al., 2015). These models are committed to symbolic representations, but they also employ probabilistic inductive learning algorithms that support graded and noisy inferences. That is, these models depart from the use of subsymbolic representations in connectionist models, and they also depart from traditional symbolic models in acknowledging that learning is probabilistic and noisy (Piantadosi & Jacobs, 2016; Tenenbaum et al., 2011).

The Bayesian framework. The Bayesian approach to the study of human cognition has generated much fruitful research in the last two decades (see Chater & Oaksford, 2008; Griffiths et al., 2010; Tenenbaum et al., 2011 for reviews). This approach has also been championed in the study of cognitive, language, and social development. Perfors, Tenenbaum, Griffiths, and Xu (2011) provide a nontechnical introduction for this approach, covering the basics of the computational and conceptual underpinnings. The inductive learning problem in cognitive development is often framed as follows: How do young human learners acquire so much knowledge so fast, given the limited amounts of data and evidence from the environment? Children are avid word learners—a child before entering first grade (and before learning to read) has a vocabulary of about 6,000 words (Bloom, 2002; Carey, 1982; Markman, 1989); children are experts in inducing the rules of grammar for their native language, in the absence of explicit instruction (Chomsky, 1987; Pinker, 1984, 1989); children construct intuitive theories during the preschool years, with little help from adults or books on science (Carey, 1985; Wellman & Gelman, 1992); children sometimes acquire biological knowledge much faster than adults—some children are experts on dinosaurs by age 4, whereas some adults only know a few dinosaur names and facts. The Bayesian framework provides a tool for understanding three key parts of inductive learning: (a) How do children make inductive inferences from just a few examples? (b) How do children acquire learning biases and inductive constraints from initial input that help them acquire knowledge more efficiently later in development? (c) How do children learn inductive frameworks and construct intuitive theories?

The basic formal tool for Bayesian models is Bayes' Rule, which captures the logic of belief revision in an elegant way:

$$p(h|X) = \frac{p(X|h)p(h)}{\sum_{h' \in H} p(X|h')p(h')}$$

To compute degrees of beliefs as probabilities depends on two components. One is the prior probability p (h), which captures how much we believe in Hypothesis h before observing any data. The other is the likelihood p (X|h), which captures the probability of observing the data X if h were true. These two components combine to yield the posterior probability, which tells us how probable h is the true hypothesis given the data p(h|X). The denominator provides a normalization term, which is the sum of the probability of each of the possible hypotheses under consideration. These probabilities of all possible hypotheses add up to 1.0, and this term accords with our intuition that if we strongly believe in one hypothesis (it has a high posterior probability) then we are less likely to believe in other competing hypotheses (they have low posterior probabilities).

To illustrate with a concrete example, we will focus on the case of learning words at different levels of a hierarchy (Xu & Tenenbaum, 2007a). The problem at hand is that although it is well documented that young children tend to first learn count nouns that refer to basic-level object categories (e.g., dog, cow, carrot; Markman, 1989; Rosch, Mervis, Gray, Johnson, & Boyes-Braem, 1976), children also face the challenge of learning the meaning of words that refer to broader categories (superordinate-level, e.g., animal) as well as those that refer to narrower categories (subordinate-level, e.g., poodle). A series of experiments with adults and preschoolers manipulated the number of labeled examples (one vs. three) and the perceptual span of the three examples (e.g., subordinate-level: three Dalmatians; basic-level: three dogs of different breed; superordinate-level: three animals from different basic-level categories, say a dog, a bird, and a fish). After being taught the novel word, "See this? It is a zav," participants were then asked to generalize the novel word to other objects in the set. The results showed that learners were sensitive to "suspicious coincidences." That is, when given one Dalmatian, adult learners generalized to the basic-level category of all dogs and child learners showed more graded generalization beyond the subordinate-level category of Dalmatians; when given three Dalmatians, however, adult and child learners restricted their generalization to only other Dalmatians, showing a sharper generalization function. Our explanation is that when a set of animals (including dogs of different breeds) and other objects are present, and the experimenter chose to label three Dalmatians, the learner reasoned that if the word "zav" had meant to refer to *all dogs*, it would have been a "suspicious coincidence" that first three labeled exemplars were all Dalmatians. Therefore they inferred a narrower extension for the novel word, and only generalized to other Dalmatians. The Bayesian model, with a preestablished hierarchy of categories that included subordinate-level, basic-level, and superordinate-level categories as distinct clusters (the prior), and a way of taking into account the number of labeled exemplars and their perceptual span (the likelihood), captured these results well. Additional experiments and modeling showed that both adult and child learners were also sensitive to whether a knowledgeable person had labeled the exemplars, and made their inferences accordingly (Xu & Tenenbaum, 2007b).

Other researchers have found evidence that supports the idea that these inductive learning mechanisms (e.g., noticing "suspicious coincidences") are domain general. For example, Gweon, Tenenbaum, and Schulz (2010) found that given one blue noise-making object, toddlers make broader generalizations than when given three blue noise-making objects sampled out of a box of blue and yellow objects. This study further demonstrated that the intention of the person while sampling objects from a population also constrains the inferences toddlers make. Nichols and colleagues (Ayars & Nichols, 2017; Nichols, Kumar, Lopez, Ayars, & Chan, 2016) found that in the moral domain, adult participants are sensitive to "suspicious coincidences" in how far they would generalize a moral rule.

The second key issue that the Bayesian framework has deepened our understanding of is the idea of "learning to learn" or overhypothesis formation (Goodman, 1955; Kemp, Perfors, & Tenenbaum, 2007). With the technical tools of hierarchical Bayesian models, research has shown that learners may use the initial input to extract regularities that become learned biases that guide later learning. A well-known example in the cognitive and language development literature is the shape bias: Young children tend to generalize novel count nouns to objects that share the same shape, but not the same texture or color (Landau, Smith, & Jones, 1988; Smith, Jones, Landau, Gershkoff-Stowe, & Samuelson, 2002, among others). One explanation of this phenomenon is that shape is indicative of kind membership (Dewar & Xu, 2009; Diesendruck, Markson, & Bloom, 2003). Many have suggested that the shape bias is learned, and an elegant training study has demonstrated that toddlers can be trained to acquire the shape bias earlier (Smith et al., 2002). If toddlers are given evidence in the lab that novel count nouns refer to sets of objects with distinct shapes, they generalize this learned bias beyond the training in the lab. A hierarchical Bayesian model captures this phenomenon by making inferences simultaneously at multiple levels (Kemp et al., 2007):

At a lower level, learners and the model infer, based on a few examples, that the count noun "ball" is used to refer to spherical objects, "cup" is used to refer to all cup-shaped objects, and so forth. At a higher-level, learners and the model infer that among a set of potential perceptual dimensions such as shape, size, texture, and color, shape appears to be the one that applies to all lower-level categories. Even with little data at the lower level, a second-order generalization, or overhypothesis, can be inferred across object categories. This overhypothesis then guides future word learning of object categories. Empirical studies also suggest that even infants are capable of forming second-order generalizations based on limited amounts of data (e.g., Dewar & Xu, 2010). Overhypothesis formation, as has been argued, provides an important tool for young learners to build larger conceptual structures and intuitive theories (Perfors et al., 2011).

These are just a few examples that demonstrate the utility of probabilistic Bayesian models in developmental research. Many domains and many aspects of learning have been investigated using a combination of behavioral experiments and computational modeling. This fast-growing body of work shows that these two types of methods enhance each other: The development of formal models may provide rational analyses at the computational (and sometimes algorithmic) level (Anderson, 1990; Griffiths et al., 2010; Griffiths, Lieder, & Goodman, 2015; Griffiths, Vul, & Sanborn, 2012; Marr, 1982; Tenenbaum et al., 2011); the models make empirical predictions that are tested in behavioral experiments, and in turn, the experimental results will inform and refine the formal models. Adopting this general approach, many labs and research groups have investigated domains such as physical reasoning, word learning, causal induction, social cognition, moral judgment, property induction, number word acquisition, grammar learning, attention allocation, pedagogy and selective trust, theory of mind, among others (e.g., Baker, Saxe, & Tenenbaum, 2009; Bonawitz, Denison, Gopnik, & Griffiths, 2014; Chater & Oaksford, 2008; Denison, Bonawitz, Gopnik, & Griffiths, 2013; Frank & Goodman, 2012; Gopnik et al., 2004; Gopnik & Schulz, 2004; Gopnik & Wellman, 2012; Griffiths et al., 2010; Gweon & Schulz, 2011; Gweon et al., 2010; Kemp & Tenenbaum, 2008; Kidd, Piantadosi, & Aslin, 2012; Kushnir & Gopnik, 2005; Liu, Ullman, Tenenbaum, & Spelke, 2017; Nichols et al., 2016; Perfors et al., 2011; Piantadosi, Tenenbaum, & Goodman, 2012; Shafto, Goodman, & Frank, 2012; Sobel, Tenenbaum, & Gopnik, 2004).

Where do hypotheses come from? It is important to note that the Bayesian approach is not without critics (see Jones & Love, 2011 and Marcus & Davis, 2013 for a rebuttal). A key open question that has been raised over and over again, in discussing the utility of the Bayesian framework for understanding cognition and development, is where hypotheses come from. Many Bayesian models assume a fixed, large hypothesis space that learners will consider, and given data, the learners will simply choose among this set of hypotheses by computing the posterior probabilities. This construal may be particularly problematic for those of us interested in development: It seems extremely implausible that children work with a fixed hypothesis space, and in fact, much evidence suggests the contrary (e.g., Siegler, 1996; Klahr, 2000; Carey, 2009). Perfors et al. (2011) offers one answer to this challenge. It may be the case that the child has the capacity to generate an infinite number of hypotheses, given a set of primitives and a set of procedures for building more complex hypotheses from these primitives. However, at any given time, this latent hypothesis space needs to deliver an explicit hypothesis space for any domain of learning, depending on the child's level of development. Then the computational and empirical question is to figure out how a learner comes up with an explicit hypothesis space such that she considers the relevant hypotheses for a particular learning task. Recent research has begun to address this issue (e.g., Bramley, Rothe, Tenenbaum, Xu, and Gureckis, 2018; Schulz, 2012; Ullman, Goodman, & Tenenbaum, 2012), employing ideas such as stochastic search, or a combination of a top-down grammar that is able to generate many hypotheses and a bottom-up procedure that takes into account the existing evidence.

There is also a more radical sense of hypothesis generation, which is critical for genuine conceptual change. Here we are concerned with the case where new primitives/new concepts may be added to the hypothesis space. I suggest that constructive thinking mechanisms may provide the key (see The Child as an Active Learner section below).

Probabilistic reasoning and other learning mechanisms in infants. The last section focuses on the idea that the use of probabilistic models has broadened our investigation of developmental phenomena and deepened our understanding of learning mechanisms. This set of ideas also dovetailed with, and inspired, much work on probing infants' earliest mechanisms of learning. For example, Saffran et al. (1996) demonstrated for the first time that 8-month-old infants, in an artificial language learning experiment, were able to track the transition probabilities of adjacent syllables in order to segment continuous speech into potential words. Aslin et al. (1998) then reported a critical experiment showing that it was indeed transition probabilities, not mere frequencies of syllable combinations, that infants kept track of. Subsequent research followed up on this line of inquiry and found that infants also keep track of transition probabilities of tones and visual forms (e.g., Kirkham, Slemmer, & Johnson, 2002). Another line of research using similar methods also showed that not only do infants track the statistics in a speech stream, they are also able to extract variables and recognize abstract patterns such as ABA or AAB from sequences of syllables (Marcus, Vijayan, Bandi Rao, & Vishton, 1999).

Studies from the last decade zoomed in on whether infants are capable of rudimentary probabilistic reasoning. Teglas, Bonatti, and their colleagues have provided compelling evidence that 12-month-old infants have intuitions about how probable an object will fall out of a "lottery machine" given some information about the population (e.g., one yellow and three blue objects bouncing around). Infants looked reliably longer at a low-probability event (e.g., the yellow object exiting the machine) than a high-probability event (e.g., one of the blue objects exiting the machine; Téglás et al., 2007, 2011). Other studies investigated infants' probabilistic intuitions with different methods. For example, Denison, Xu, and their colleagues report that 6- and 8-month-old infants (but not 4-month-olds) look longer at a low-probability outcome than a high-probability outcome when samples are drawn randomly from a box of red and white Ping pong balls (Denison, Reed, & Xu, 2013; Denison, Trikutam, & Xu, 2014; Xu & Garcia, 2008). Furthermore, these computations are strong and robust enough to support prediction and action—10- to 14-month-old infants crawl to a cup that is more likely to contain a preferred object based on probabilities of a random draw (Denison & Xu, 2010b, 2014; see Denison & Xu, in press, for a review). A number of studies also demonstrated that these mechanisms are inferential in the sense that they take into account substantive constraints in both the physical and psychological worlds, and the infant learners are sensitive to violations of random sampling in how they make inferences (e.g., Denison & Xu, 2010a; Kushnir, Xu, & Wellman, 2010; Téglás et al., 2007, 2011; Xu & Denison, 2009; Wellman, Kushnir, Xu, & Brink, 2016). Lastly, in an elegant recent study, Cesana-Arlotti et al. (2018) found that in addition to these probabilistic reasoning abilities, 12- to 19-month-old infants' reasoning toolbox may also include precursors to deductive reasoning.

This body of research provides compelling evidence that infants are not only good at keeping track of frequencies and correlations (i.e., associative learning mechanisms), they also have a set of sophisticated statistical inferential mechanisms that support inductive learning from at least 6 months onward. These powerful rational, statistical, and inferential mechanisms allow human infants to acquire knowledge rapidly and accurately across many domains (see Xu & Kushnir, 2013, 2012 for reviews). Their existence plays a key part in explaining young children's rapid cognitive growth within the first few years of life in the absence of much formal instruction—a feat that astonishes most adults.

Constructive Thinking as Mechanisms for Genuine Conceptual Change and Hypothesis Generation

Belief revision is a key part of learning—it happens every day in all domains of knowledge—and the Bayesian framework provides a normative standard for rational updates of beliefs. These beliefs may be at the level of individual facts or at the level of overhypotheses or intuitive theories. For the most part, the Bayesian inductive learning mechanisms have been applied in situations in which the learner already has a set of relevant hypotheses in mind, and in light of evidence, the learner updates the posterior probabilities of each of the hypotheses. The evidence the learners use may come from various sources, for example, data generated by the learner herself, observations, or via testimony. However, not all learning is belief revision (contra Fodor, 1980). Genuine conceptual change is both possible and actual (Carey, 1985, 1991, 2009; Gopnik & Meltzoff, 1997; Wellman & Gelman, 1992).

Following Gendler (2000); Lombrozo (2012), and others, here I argue that *constructive thinking* mechanisms (also known as "learning by thinking"; Lombrozo, 2018) support radical conceptual change, because they are hypothesis generation mechanisms that may deliver something genuinely new. This suite of mechanisms includes analogy, explanation, mental imagery, mental simulation, and thought experiment. All these mechanisms have been studied in the history and philosophy of science, and also in cognitive and developmental psychology. Given the scope of this paper, I will give just a few examples of these cognitive processes to illustrate their importance.

Thought Experiments

Thought experiment is often considered the paradigmatic case of "learning by thinking" in philosophy of science (e.g., Gendler, 1998, 2000). Galileo's famous thought experiment of dropping different size objects from the Tower of Pisa gives historians and philosophers a clear case of how thought experiments may deliver something genuinely new, and how philosophers and cognitive scientists may understand how this kind of "discovery" is possible (see Gendler, 2000, for a detailed analysis). In the cognitive development literature, an elegant case study on children's conception of weight, density, size, and material kind shows that children could also reason through a set of premises and arrive at the correct logical conclusion even though they may not ever obtain the relevant empirical observations (Smith et al., 1985). This case study probes 3- to 9-year-olds' understanding of how these concepts about material kind are embedded in a theory-like structure, and how an undifferentiated concept of weight/density gets differentiated. During the preschool years, children's concept of weight is "felt weight"—if an object is put on your hand and you can feel its weight, then it is material and has mass. With this conception of weight, children will often answer that one grain of rice or a very small piece of Play-Doh or Styrofoam weighs "nothing at all." By age 8 or 9, however, children come to realize that all objects and material kinds, no matter how small, weigh something. Part of their reasoning is a thought experiment: A child knows that a pile of grains of rice has weight. If one makes the pile smaller, the weight will be less. If one makes the pile smaller still, the weight will be even less. Eventually when there is only one grain of rice left, the child thinks it weighs nothing. But, when one puts more and more grains of rice together, the pile will start to weigh something. How is it possible that if one grain of rice weighs nothing, but 20 or 100 grains of rice start to weigh something? By getting herself into a contradiction, the child realizes that even though one grain of rice does not feel like it has weight in her hand, it nevertheless has to weigh a tiny bit. At this point, the child's concept of weight has changed from "felt weight" to a more accurate concept that all material kinds have weight. This is a thought experiment because the child did not acquire this piece of knowledge by getting a more sensitive scale to see if one grain of rice really weighs something. Instead she works through the logic and draws the correct inference that the concept of weight is different from the concept of "felt weight," and as she follows rational principles of reasoning, she learns that even a tiny piece of any material kind must weigh something.

Explanation

Many developmental psychologists and cognitive scientists have argued for a privileged role of explanation in understanding development (Chi, Bassok, Lewis, Reimann, & Glaser, 1989; Chi, de Leeuw, Chiu, & LaVancher, 1994; Keil, 2006; Lombrozo, 2006, 2016; Wellman, 2011, among others). Lombrozo and her colleagues have studied the effects of explanation extensively in recent years, and they have articulated a set of *explanatory virtues* that help us understand why explanation provides constraints on the reasoning process. For example, one study found that adults are more likely to find broad patterns in a set of data when asked to explain than when asked to describe (Williams & Lombrozo, 2012). This tendency is so strong that in a later study, they found that explaining impaired learning when there were no broad patterns to be found in the data presented to participants (Williams, Lombrozo, & Rehder, 2013). They dubbed this explanatory virtue the subsumptive constraint. Similarly, when asked to explain, both adults and children prefer simpler causes as opposed to more complex ones (e.g., invoking one cause for two instances as opposed to positing a distinct cause for each instance) when asked to explain than in a control condition (Walker, Bonawitz, & Lombrozo, 2017). Thus, simplicity is another explanatory virtue. Lombrozo (2018) has suggested that the epistemic force of explanation as a learning-by-thinking mechanism is that ". . . the process of engaging in explanation recruits explanatory virtues as evaluative criteria, and these in turn act as constraints on learning and inference by leading learners to seek and privilege hypotheses that support these virtues." It is impressive that even preschoolers behave in a way that accord with these explanatory virtues, with little formal instruction as far as we know. Explanation, seen in this light, provides a source of new hypotheses that do not solely rely on the data themselves.

Analogy

Analogical reasoning is another extensively discussed mechanism for hypothesis generation and scientific change (Holyoak, 2012). Holyoak and Thagard (1989) suggests that "analogical inference—using a source analog to form a new conjecture, whether it be a step toward solving a math problem, a scientific hypothesis, a diagnosis for puzzling medical symptoms, or a basis for deciding a legal case—is the fundamental purpose of analogical reasoning" (p. 128). One important procedure for analogical reasoning is *structural mapping* (Gentner, 1983) and finding "alignable differences" (Markman & Gentner, 1993). The idea is that the learner examines the structure of the source domain, and for each element, tries to identify the corresponding element in the target domain. Critically, when there is not a corresponding element, postulating one as needed—that is, a new concept or novel hypothesis.

Recent studies have found that even 3- to 9-month-old infants have some rudimentary capacity for analogical reasoning. Infants use the process of comparison to extract nonobvious commonalities across sets of exemplars, for example, same versus different, and generalize accordingly on test trials (Anderson, Chang, Hespos, & Gentner, 2018). This finding fits nicely with the idea that infants can extract variables—an impressive ability for abstraction—while ignoring more obvious and salient perceptual features (e.g., Marcus et al., 1999). Gentner and Hoyos (2017) further suggests that analogical reasoning may be the underlying mechanism for overhypothesis formation, which, as suggested in a previous section, plays a critical role in developing learning biases and constructing larger conceptual structures.

Preschoolers use the process of comparison to generate new hypotheses. In an elegant study, Christie and Gentner (2010) found that structural alignment helps 3- and 4-year-old children to move away from their default mode of generalizing based on object similarity to generalizing based on relational similarity.

These and many other studies support the claim that the ability for analogical reasoning—a mechanism for hypothesis generation—develops rapidly during the first few years of life, and they play a crucial role in building intuitive theories of various content domains.

In sum, constructive thinking ("learning by thinking"; Gendler, 2000; Lombrozo, 2018) allows the learner to construct novel explanations, imagine alternative scenarios and possible worlds, and generate analogies across domains. This suite of mechanisms, generally speaking, enlarge a mind's conceptual repertoire by going beyond the data and generating novel ideas and solutions that may increase the coherence and elegance of a current theory or overturn it in favor of a new theory.

The Child as an Active Learner

The last tenet of the rational constructivist view grows out of an old idea in constructivist theories of development (Bruner, 1961; Piaget, 1954). This is an important part of a theory of cognitive development because the child faces a world full of information and potential evidence for belief updating and theory building, and she needs to have a "theory of evidence" that helps sift through the enormous amount of information in the environment, both in data processing and in data generation (Fedyk, Kushnir, & Xu, in press).

The "child as an active learner" has been an enduring theme in the study of developmental psychology. The basic intuition is that children are not just passive recipients of input from the environment; instead they play an active role in their own development. Bruner (1961) strongly advocated for an active learning model for education, based on ideas from developmental psychology. Piaget (1954) documented in great detail how his own children manipulated objects and he used these observations for developing an account of how object permanence is acquired through a child's active interventions in the world. More recently, several research groups have resurrected this idea (Gopnik & Bonawitz, 2015; Gopnik & Wellman, 2012; Kidd & Hayden, 2015; Kidd et al., 2012; Schulz, 2012; Singer et al., 2006; Xu & Kushnir, 2013, among others). In particular, Schulz (2012) argued that the way children acquire knowledge about the world is similar to the various epistemic practices we observe in scientists, extending the analogy between learning in childhood and scientific theory building. This account suggests that like scientists, young children selectively explore evidence that is confounded and unexpected, isolate candidate causes in order to decide between hypotheses, and make rational decisions about when to rely on others' knowledge.

Recent research on the active child learner has focused on two issues that are critical for developing accounts of active learning. One set of studies has focused on infants and children's attention allocation, curiosity, interest, and information seeking (Begus, Gliga, & Southgate, 2014, 2016; Gerken, Balcomb, & Minton, 2011; Goupil, Romand-Monnier, & Kouider, 2016; Gruber, Gelman, & Ranganath, 2014; Kidd & Hayden, 2015; Kidd et al., 2012; Stahl & Feigenson, 2015); with preschoolers and older children, various studies have focused on whether young children can generate informative data on their own, be it in the form of causal intervention or question-asking (Bonawitz, van Schijndel, Friel, & Schulz, 2012; Legare, 2012; Legare, Gelman, & Wellman, 2010; McCormack, Bramley, Frosch, Patrick, & Lagnado, 2016; Ruggeri, Lombrozo, Griffiths, & Xu, 2016; Sim & Xu, 2017). Much of the work has also centered on the issue of whether infants' and children's behaviors can be explained by measures of uncertainty (e.g., Coughlin, Hembacher, Lyons, & Ghetti, 2015; Vredenburgh & Kushnir, 2016), and whether formal computational models may help us understand the computational underpinnings of these phenomena (e.g., Coenen, Rehder, & Gureckis, 2015; Kidd et al., 2012; McCormack et al., 2016; Meng, Bramley, & Xu, 2018; Ruggeri et al., 2016, 2017). Some studies have also investigated systematically whether active learning is superior to other forms of learning, in both adults and children (e.g., Markant & Gureckis, 2014; Markant, Ruggeri, Gureckis, & Xu, 2016; Ruggeri et al., 2016; Sim & Xu, 2017).

Starting in the second half of the first year, infants are not simply drawn to salient aspects of their environment; they begin to allocate their attention differentially depending on whether there are potential learning opportunities or not. For example, 8-month-old infants remain attentive when facing a sequence of objects that is neither too predictable nor too unpredictable, as if to say that they choose to attend to stimuli that are "just right." Furthermore, infants' looking away behavior (an index of selective attention) is well captured by a probabilistic Bayesian model, even at the level of individual infants (Kidd et al., 2012; Piantadosi, Kidd, & Aslin, 2014). In an artificial language learning study, Gerken et al. (2011) found that 11-month-old infants stop attending to speech streams that appear to contain nonlearnable rules of natural language. Stahl and Feigenson (2015) showed that when infants witness an object violating a physical rule (e.g., that a solid object cannot pass through another solid object), they pay more attention to the violation object and manipulate the object to reproduce the surprising effect. Furthermore, these infants learn an arbitrary property faster when it is taught on the violation object than a nonviolation object.

With toddlers, Sim and Xu (2017) found that 13-month-olds choose to crawl toward and play with a box that had yielded a "suspicious" sequence of seemingly random draws of ping pong balls. Begus et al. (2014) found that 14-month-olds' pointing behavior may be indicative of information seeking—when taught a new property on the object that they had pointed to, they learned better than if the property had been taught on an object that they had not pointed to. Goupil et al. (2016) found that at 20 months, infants can monitor their own uncertainty about the location of a hidden object, and seek help from others to improve their chance of retrieving the object.

With 2- and 3-year-old children, there is some evidence that they can generate informative data for themselves. In a causal learning task, Sim and Xu (2017) found that children in a free play condition (in which they were given the opportunity to figure out on their own which blocks make the toy machines play music; cf. Gopnik & Sobel, 2000) did just as well on the test trials as children in a didactic condition (in which they were shown evidence by an experimenter which blocks activate the toy machines). Interestingly, this was not the case with 19-month-olds, who needed the help of the parents and experimenters in the same task. Schulz, Gopnik, and Glymour (2007) found that preschoolers sometimes generate effective data to tease apart alternative hypotheses in causal learning. With preschoolers, there is also some preliminary evidence that when learning a complex rule, free play may outperform didactic demonstrations (Sim, Mahal, & Xu, 2017).

Recent research on active learning in adults has informed a great deal of developmental research on this topic. Gureckis, Markant, and their colleagues have used a yoked design to probe whether active learning really confers a learning advantage (Gureckis & Markant, 2012; Markant & Gureckis, 2014), and found that adults, when given the opportunity to test their own hypotheses in an active learning condition, consistently outperform those in a yoked passive condition in which the participants received the same data. Castro, Kalish, and Nowak (2008). Also discovered that adults perform better in an active learning condition, and their strategy is closer to the optimal strategy. Similar findings have been obtained with 7- to 10-year-old children (Ruggeri et al., 2016; Sim, Tanner, Alpert, & Xu, 2015).

An overall picture has begun to emerge that helps us characterize active learning in children. With infants, there is now a wealth of evidence showing that their attention allocation is based on some assessment of learning potential, and following infants' indication of curiosity or interest may enhance learning. With infants, however, it is unclear if they are able to generate their own data effectively. With preschoolers, empirical studies have shown, convincingly, that they are able to generate informative data on their own, although at the moment it is unclear whether the self-generation of evidence follows any rational principles (McCormack et al., 2016; Meng et al., 2018).

The child-as-an-active-learner claim captures the idea that development is an interplay between the child and her environment. Even as infants, the young learner chooses what to attend to, what to play with, who to learn from, and where to seek help. As the learner grows, she also learns to generate her own data, directly (as in manipulating the physical world) or indirectly (as in seeking explanations through question asking). In other words, *cognitive agency* is an integral part of a theory of development (see Fedyk & Xu, 2018 for discussion).

Some Corollaries and Implications of the Rational Constructivist Approach

Why Is Rational Constructivism Not a Piagetian View

Three reasons make the rational constructivist theory a non-Piagetian view. First, our characterization of the initial state departs radically from Piaget's as well as other empiricists' view. The newborn human infant is not in a "blooming buzzing confusion"—far from it. The newborn infant is highly competent in two senses: She begins life with a set of proto-conceptual primitives that remain functional throughout her life, and these primitives are computationally and inferentially complex; she also begins life with a set of powerful rational, statistical, and inferential learning mechanisms that propels development forward rapidly from infancy on. Second, contra to Piagetian stage theory, infants already have the capability for thinking in symbols (especially as language learning progresses in a fast pace from birth, or even in vitro) and forming abstractions (in the form of rudimentary analogical reasoning and variable abstraction). Infants' knowledge about the world is infinitely richer than what Piaget had posited for the helpless newborn, and development cannot be divided into stages that are qualitatively different from each other. Instead, learning and development are driven by language and symbol learning, rational relief revision, and constructive thinking for genuine conceptual change. These mechanisms operate in parallel throughout development. Third, the types of learning mechanisms that we have discussed here depart radically from the Piagetian idea of "logical construction"—that the child progresses through a set of stages with increasing logical capacities (e.g., from not being able to think about superset/subset relations to having such an ability, Flavell, 1963). Yet I also hope that it is clear from the preceding discussion that rational constructivism is a *constructivist* theory of development. The young human learner actively engages in the learning process from infancy onward, and she *constructs* new concepts, new learning biases, new beliefs, and new intuitive theories that may be radically different from what she is born with. Furthermore, although infants may start life with rather complex representations that are domain specific, the learning mechanisms I have discussed are all domain-general ones. These domain-general mechanisms drive development by changing the format of the initial representations that lead to the construction of domain-specific intuitive theories, by providing a rational way for belief revision, and by constructing new ideas and new hypotheses to engender genuine conceptual change.

The Utility of Formal Computational Models in Understanding Learning and Development

The rational constructivist view is largely inspired by the surge of research on using probabilistic models (mostly of the Bayesian variety) in studying human cognition and learning. In the last decade or so, the Bayesian approach has generated much interesting, important, and groundbreaking computational work (e.g., in terms of developing mathematical tools to formalize important insights and phenomena in cognitive science, and developing new algorithms for understanding process-level mechanisms) and empirical findings (e.g., in terms of understanding the importance of compositionality and systematicity of thought, semantics, probabilistic reasoning mechanisms, the tradeoff between heuristics and normative Bayesian reasoning; see Chater & Oaksford, 2008; Griffiths et al., 2010; Tenenbaum et al., 2011 for reviews). This has been an incredibly fruitful and productive research enterprise. A group of developmental psychologists have built long lasting collaborative partnerships with computational cognitive scientists, and these collaborations have led to new ways of thinking about learning and development (e.g., see Xu & Griffiths, 2011; Xu & Kushnir, 2013, 2012 for reviews). Taking formal modeling seriously has shed new light on fundamental issues such as nature versus nurture, innate versus learned, and mechanisms of learning and developmental change.

Here I have emphasized the utility of Bayesian probabilistic models in understanding cognitive development. But of course a variety of computational models have been developed over the last three decades that aim to shed light on developmental processes (see Marcovitch & Zelazo, 2012). As Schlesinger and McMurray (2012) put succinctly, the contributions of computational models are many-fold. Models often make researchers be more explicit about their theoretical commitments; models help us spell out causal mechanisms; models—both successful and failed—may provide constraints on learning; models may help bridge levels of analysis given the precision of a mathematical language; models may give us opportunities to study issues that are difficult to investigate empirically, such as critical period, sensory deprivation, atypical development, and so forth; and lastly, modeling must go hand in hand with behavioral experimentation because they naturally inform each other.

Implications for Philosophy of Science and Epistemology

Over the years, a few researchers have begun to consider the philosophical implications of thinking about belief revision within the Bayesian framework, and how the rational constructivist approach may inform the study of philosophy of science and epistemology. For example, Henderson, Goodman, Tenenbaum, and Woodward (2010) have argued that hierarchical Bayesian models provide a plausible model for understanding the structure of scientific theories—these theories consist of multiple levels, higher level theories may guide learning at the lower levels, and theory change may be captured with these computational tools.

More recently, Fedyk and Xu (2018) attempt to develop an account of rationality based on the concepts of Bayesian rationality, creative rationality, and cognitive agency. Bayesian rationality is a well-developed framework for belief revision, normatively and descriptively. However, to accommodate constructive thinking mechanisms, creative rationality is needed—young learners are often in need of generating novel ideas that go beyond the currently available data, and the balance between this sort of creativity and everyday belief revision constitutes a form of homeostasis in development. Lastly, cognitive agency firmly puts the child at the center of her learning endeavor—learning is an active, agentive, and social process. Fedyk and Xu (2018) further suggests that this type of analysis brings the study of epistemology closer to contemporary cognitive science, and it follows from Quine's call for a "naturalized epistemology" (Quine, 1969).

Future Directions

In conclusion, rational constructivism aims to go beyond the nativism versus empiricism debate, and beyond traditional Piagetian constructivist theory of development. Here we explicate the central tenets of this new theoretical framework, in the hopes of generating new theoretical discussions and new empirical investigations.

Many questions remain open given the proceeding discussion on rational constructivism. Here I focus on four key issues for future research. First, if initial representations and later intuitive theories require distinct representational formats, what are they exactly and how can we capture the differences in computational terms? This issue is intimately related to the question of how (and whether) language transforms initial representations into the format of a language of thought, such that learners can then formulate intuitive theories in propositional attitudes (Carey & Spelke, 1996; Gopnik, 1996; Karmiloff-Smith, 1990). Furthermore, is learning a natural language instrumental for a learner's thoughts to be compositional, systematic, and productive in the Fodorian sense (Fodor, 1975)?

Second, on the computational side, modeling efforts have shed new light and inspired new empirical work across many domains of cognition and development. But so far most of the models have focused on the computational level of analysis (Marr, 1982). How can we go beyond computational level analysis and get down to the nitty-gritty of developing algorithms that capture real human behavior (Griffiths, Lieder, & Goodman, 2015)? A systematic investigation of this issue may also shed light on the nagging question of how to reconcile the Bayesian learner with one who falls prey to heuristics and biases. If we are correct in claiming that infants are intuitive statisticians, do children learn to employ heuristics and biases over the course of development in order to rationally budget cognitive resources, as some have argued in recent years (e.g., Gualtieri & Denison, 2018; Lieder & Griffiths, 2017)?

Third, what is the tradeoff between employing Bayesian inductive mechanisms and constructive thinking (or "learning by thinking") mechanisms? The former provides a computationally rigorous framework for belief revision, and much learning is about belief revision, for both adults and children. Yet we also know that learning and development is not just about belief revision. Intuitive theories, which constitutes larger conceptual structures, sometimes undergo radical conceptual change themselves, and sometimes new domain theories are formed (e.g., intuitive chemistry, Au, 1994; intuitive astronomy, Vosniadou & Brewer, 1994). If a child is always in a belief revision mode, she can only fine-tune her existing beliefs since she has a fixed hypothesis space to work with. How does she generate new ideas and novel hypotheses? One answer is that "learning by thinking" mechanisms (Gendler, 2000; Lombrozo, 2018) work together with belief revision mechanisms to balance when to pursue radical new ideas and when to tweak existing beliefs. This, at the moment, is a wide open question, and a real challenge for further theoretical, empirical, and computational inquiry.

Lastly, what are the limits of the child as an active learner, and would understanding these limits revise our characterization of children's learning? Despite the wide consensus that infants and young children play an active role in their own development, we still lack a coherent account of what it means. It appears that the inquisitive child is curious and exploratory, but she may not be effective in actually generating her own data. Because a large part of advancing science—with the ultimate goal of building more and more accurate causal models of the world—is about generating informative data by following the scientific method, should we revisit the idea that the child is a little scientist (e.g., D. Kuhn, 1989)?

These and many other questions are ripe for further theoretical and empirical investigations, and I have no doubt that the field of cognitive development will make great strides in answering these questions in the years to come.

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